First Semester B.E. Degree Examination, June/July 2017
Engineering Mathematics – I

Time: 3 hrs.  Max. Marks: 100

Note: Answer any FIVE full questions, selecting at least ONE question from each part.

Module-1

1. a. Find the n\textsuperscript{th} derivative of \( y = \sin^2 x \sin h^2 x + \log_{10} (x^2 - 3x + 2) \). (07 Marks)
b. Find the pedal equation for the curve \( r = a + b \cos \theta \). (06 Marks)
c. Obtain radius of curvature for the parametric curve, \( x = a(\sin t) \), \( y = a(1 \cos t) \). (07 Marks)

2. a. If \( y = \tan^4 x \), prove that \((1 + x^2)y_{n+2} + 2(n + 1)xy_{n+1} + n(n + 1)y_n = 0 \). Hence obtain \( y_n(0) \). (07 Marks)
b. Find the angle of intersection between the curves \( r = 2 \sin \theta \); \( r = 2(\sin \theta + \cos \theta) \). (06 Marks)
c. Find the radius of curvature for the polar curve \( r^2 = a^2 \cos 2 \theta \). (07 Marks)

Module-2

3. a. Evaluate \( \lim_{x \to 0} (\cos x)\cot^2 x \). (06 Marks)
b. Determine Maclarin’s series for the function for \( f(x) = \log (1 + \cos x) \) upto term containing \( x^4 \). (07 Marks)
c. If \( u = f(2x - 3y, 3y - 4z, 4z - 2x) \) then obtain the value of \( \frac{1}{2} \frac{\partial u}{\partial x} + \frac{1}{3} \frac{\partial u}{\partial y} + \frac{1}{4} \frac{\partial u}{\partial z} \). (07 Marks)

4. a. Find total derivative of \( u \) with respect to \( t \) where \( u = \tan^{-1} x/y, x = e^t - e^{-t}, y = e^t + e^{-t} \). (06 Marks)
b. If \( u = \frac{x}{y - z}, v = \frac{y}{z - x}, w = \frac{z}{x - y} \), find the Jacobian \( \frac{\partial (u, v, w)}{\partial (x, y, z)} \). Determine whether \( u, v \) and \( w \) are functionally dependent. (07 Marks)
c. If \( x, y, z \) be the angles of a triangle, show that the maximum value of \( \sin x \sin y \sin z \) is \( \frac{3\sqrt{3}}{8} \). (07 Marks)

Module-3

5. a. A particle moves along \( x = t^3 - 4t, y = t^4 + 4t, z = 8t^2 - 3t^3 \), where ‘\( t \)’ denotes time. Find the magnitudes of velocity and acceleration at time \( t = 2 \). (07 Marks)
b. Assuming the validity of differentiation under integral sign prove that \( \int_{0}^{\infty} e^{-x^2} \cos x \alpha dx = \frac{\sqrt{\pi}}{2} e^{-\alpha^2/4} \). (07 Marks)
c. Trace the curve \( x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}} \), using general rules of tracing the curve. (06 Marks)

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6. a. If \( \vec{F} = \nabla (x^3 + y^3 + z^3 - 3xyz) \) find \( \text{curl} \vec{F} \). Is \( \vec{F} \) irrotational? (07 Marks)

b. Prove that if \( \vec{F} \) is a vector point function \( \text{div} (\text{curl} \vec{F}) = 0 \). (07 Marks)

c. If \( \vec{r} \) is a position vector of a point in space obtain \( \text{div} \vec{r} \) and \( \text{curl} \vec{r} \). (06 Marks)

**Module-4**

7. a. Solve \( \frac{dy}{dx} + x \sin 2y = x^3 \cos^2 y \). (07 Marks)

b. Obtain the reduction formula for \( \int_0^{\pi/2} \cos^n x \, dx \), where ‘n’ is a positive integer. (07 Marks)

c. A body originally at 80°C cools down to 60°C in 20 minutes, the temperature of air being 40°C. What will be the temperature of the body after 40 minutes from the original? (06 Marks)

8. a. Show that family \( \frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1 \) with \( \lambda \) as a parameter is self orthogonal. (07 Marks)

b. Evaluate \( \int_0^a x^3 \sqrt{2ax - x^2} \, dx \). (07 Marks)

c. Solve \( (y^2 e^{xy^2} + 4x^3) \, dx + (2xye^{xy^2} + 3y^2) \, dy = 0 \). (06 Marks)

**Module-5**

9. a. Solve by gauss elimination method:
   2x – 3y + 4z = 7
   5x – 2y + 2z = 7
   6x – 3y + 10z = 23. (07 Marks)

b. Reduce the quadratic form \( 3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3 \) into canonical form by orthogonal transformation. (07 Marks)

c. Find the largest eigen value and corresponding eigen vector by Rayeligh’s power method
   performing five iterations, with \( x^{(0)} = [1, 1, 1]^T \) for \( A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \). (06 Marks)

10. a. Solve by LU decomposition method:
   10x + y + z = 12
   2x + 10y + z = 13
   2x + 2y + 10z = 14. (07 Marks)

b. Diagonalize the matrix \( A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix} \). Hence find \( A^4 \). (07 Marks)

c. Solve by Gauss Seidel iteration method:
   20x + y - 2z = 17
   3x + 20y - z = -18
   2x - 3y + 20z = 25
   Perform 3 iterations. (06 Marks)

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