Mechanics of Metal Cutting

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1. MECHANICS OF CHIP FORMATION,
A single point cutting tool may be either right or left hand cut tool depending on the direction of feed.
Material Removal Rate

\[ MRR = vfd \]

Roughing (R)
\[ f = 0.4 - 1.25 \text{mm/rev} \]
\[ d = 2.5 - 20 \text{mm} \]

Finishing (F)
\[ f = 0.125 - 0.4 \text{mm/rev} \]
\[ d = 0.75 - 2.0 \text{mm} \]

\( v_R \ll v_F \)
Tool Terminology

- Side Rake (SR), +
- Back Rake (BR), +
- Rake Face
- Flank Face
- Shank
- Turning Cutting edge
- Side relief angle
- Side cutting edge angle (SCEA)
- Nose Radius
- Facing Cutting edge
- End Cutting edge angle (ECEA)
- Clearance or end relief angle
Manufacturing Technology

Tool signature for single point cutting tool
Manufacturing Technology

Tool signature for single point cutting tool

- Shank
  - It is the main body of the tool
- Flank
  - The surface of the tool adjacent to the cutting edge
- Face
  - The surface on which the chip slides
- Nose
  - It is the point where the side cutting edge and end cutting edge intersect
- Nose Radius
  - Strengthens finishing point of tool
- Cutting Edge
  - It is the edge on the face of the tool which removes the material from the work piece
- Side cutting edge angle
  - Angle between side cutting edge and the side of the tool shank
Cutting Geometry

Diagram showing cutting geometry with labels:
- Chip
- Cutting tool
- Motion of tool (relative to work)
- Rake face
- Flank
- Original surface
- New surface
- Shear deformation to form chip
- Workpart
- Cutting edge of tool
- Motion of chip
- Negative rake angle
- Relief angle
- Cutting edge
Cutting Models

ORTHOGONAL GEOMETRY

OBLIQUE GEOMETRY
Orthogonal and Oblique Cutting

The two basic methods of metal cutting using a single point tool are the orthogonal (2D) and oblique (3D). Orthogonal cutting takes place when the cutting face of the tool is 90 degree to the line of action of the tool. If the cutting face is inclined at an angle less than 90 degree to the line of action of the tool, the cutting action is known as oblique.
Orthogonal and Oblique Cutting

Orthogonal Cutting:
- The cutting edge of the tool remains normal to the direction of tool feed or work feed.
- The direction of the chip flow velocity is normal to the cutting edge of the tool.
- Here only two components of forces are acting: Cutting Force and Thrust Force. So the metal cutting may be considered as a two dimensional cutting.

Oblique Cutting:
- The cutting edge of the tool remains inclined at an acute angle to the direction of tool feed or work feed.
- The direction of the chip flow velocity is at an angle with the normal to the cutting edge of the tool. The angle is known as chip flow angle.
- Here three components of forces are acting: Cutting Force, Radial force and Thrust Force or feed force. So the metal cutting may be considered as a three dimensional cutting.
- The cutting edge being oblique, the shear force acts on a larger area and thus tool life is increased.
Assumptions
(Orthogonal Cutting Model)

- The cutting edge is a straight line extending perpendicular to the direction of motion, and it generates a plane surface as the work moves past it.
- The tool is perfectly sharp (no contact along the clearance face).
- The shearing surface is a plane extending upward from the cutting edge.
- The chip does not flow to either side.
- The depth of cut/chip thickness is constant uniform relative velocity between work and tool.
- Continuous chip, no built-up-edge (BUE)
Manufacturing Technology

Turning

Single point cutting tool removes material from a rotating work piece to form a cylindrical shape.
Mechanics of Metal Cutting

A cutting tool exerts compressive force on the workpiece which stresses the work material beyond the yield point and therefore metal deform plastically and shears off.

Plastic flow takes place in a localized region called the shear plane.

Sheared material begins to flow along the cutting tool face in the form of chips.

Flowing chips cause tool wear.

Applied compressive force which leads to formation of chips is called cutting force.

➤ Heat produced during shearing action raises the temperature of the workpiece, cutting tool and chips.

➤ Temperature rise in cutting tool softens and causes loss of keenness in cutting edge.

➤ Cutting force, heat and abrasive wear are important features in metal cutting.
MECHANISM OF CHIP FORMATION

- The fig. represents the *shaping operation*, where the work piece remains stationary and the tool advances in to the work piece towards left.
- Thus the metal gets compressed very severely, causing *shear stress*.
- This stress is maximum along the plane is called *shear plane*.
- If the material of the workpiece is *ductile*, the material flows *plastically* along the shear plane, forming *chip*, which flows upwards along the face of the tool.
When the cutting tool is forced against the work, the metal layer which is just ahead of tool is compressed.

If the tool is forced further, a condition will be reached, in which the stress exceeds ultimate shear strength of the given work material.

This leads shear along the shear plane and cutting off the chip from the workpiece.

With further movement of the tool, the new layer is compressed and the cycle is repeated.

The chip formed in the metal cutting operations, undergoes plastic deformation, it becomes shorter (chip contraction) and cross-section increases.

Due to contraction, the length of chip is shorter than the length of the tool travel, along the surface of the work.
Deformation of metal occurs along shear plane. However, in realistic model the shear deformation occurs within a shear zone (Primary shear zone).

An other shear occurs due to friction between the chip and tool as the chip slides along the rake face of the tool. This is referred as secondary shear zone.

Another shear occurs between work and tool interface, which is called as tertiary shear zone.

If machined at low cutting speed → Shear zone is thick
If machined at high cutting speed → Shear zone is thin
CHIP FORMATION

Tool will cut or shear off the metal, provided

1. Tool is **harder** than the work metal.

2. Tool is **properly shaped** so that its edge can be effective in cutting the metal.

3. The tool is **strong enough** to resist the cutting pressures.

4. **Movement of the tool** relative to the material or vice versa, so as to make cutting action possible.
TYPES OF CHIP

✓ Continuous chip
✓ Discontinuous chip
✓ Continuous chip with built-up edge
✓ Serrated chips
TYPES OF CHIPS

The chips produced during machining can be broadly classified as 3 types:

1. Continuous chips
2. Discontinuous chips or Segmental chips
3. Continuous chips with build-up edge
TYPES OF CHIP

- Continuous chip
  - A long continuous chip will result when
    - work material is ductile
    - Cutting speed is high
    - Small feed and depth of cut
    - A sharp cutting edge
    - Low tool-chip friction
  - Good surface finish results when this type of chip is formed.
  - Turning tools are often equipped with chip breakers to solve the problems of chip disposal when became long.
TYPES OF CHIP

- Discontinuous chip
  - A discontinuous chip will result when
    - Brittle work materials
    - Low cutting speeds
    - Large feed and depth of cut
    - High tool-chip friction
  - Form into separate segments as shown in fig
  - Impart an irregular texture to the machined surface
Discontinuous Chips Contd..

Stages of formation of Discontinuous chips
Continuous chips with BUE

- In machining ductile metals like steels with long chip-tool contact length (small rake angle), a lot of stress and temperature develops in the secondary deformation zone at the chip-tool interface.
- Under such high stress and temperature in between two clean surfaces of metals, strong bonding may locally take place due to adhesion similar to welding.
- In ductile materials, with lower cutting speeds, small particles of cut chip adheres, under the action of pressure and temperature, to the face of the tool.
Continuous chips with BUE

- Continuous chips with BUE are formed when machining ductile metals with a cutting tool of smaller rake angle at lower cutting speed. The other conditions which give rise to BUE are:
  1. Higher values of feed and depth of cut
  2. High friction
  3. Poor lubrication
  4. High cutting pressure and temperature in shear zone

- These BUE eventually swept from the tool and remain attached to the machined surface.

- This causes poor surface finish of work surface.

- Presence of build up edge increases power consumption.
TYPES OF CHIP

- Serrated/segmented chip
  - These chips are semi continuous in the sense that they possess a saw-tooth appearance that is produced by cyclical chip formation of alternating high shear strain followed by low shear strain.
  - These chips are associated with difficult-to-machine metals when they are machined at higher cutting speeds e.g.
    - Titanium alloy
    - Nickel-base super alloys
    - Austenitic stainless steels
    - Steel
Orthogonal Cutting

\[ r = \frac{t_w}{t_c} = \frac{l_s \sin \phi}{l_s \cos(\phi - \alpha)} \]

\[ \tan \phi = \frac{r \cos \alpha}{1 - rs \sin \alpha} \]

\[ \gamma = \frac{AC}{BD} = \frac{AD + DC}{BD} = \tan(\phi - \alpha) + \cot \phi \]
Mechanics of Orthogonal Cutting

Shear Plane Angle Proof

\[ t_1 = h \sin \theta, \quad t_2 = h \cos(\theta - \alpha) \]

\[ r = \frac{t_1}{t_2} = \frac{h \sin \theta}{h \cos(\theta - \alpha)} = \frac{\sin \theta}{\cos \theta \cos \alpha + \sin \theta \sin \alpha} \]

\[ r \cos \theta \cos \alpha + r \sin \theta \sin \alpha = \sin \theta \]

\[ \frac{r \cos \theta \cos \alpha}{\sin \theta} + \frac{r \sin \theta \sin \alpha}{\sin \theta} = 1 \]

\[ \frac{r \cos \alpha}{\tan \theta} + r \sin \alpha = 1 \]

\[ \tan \theta = \frac{r \cos \alpha}{1 - r \sin \alpha} \]
‘Turning’ Forces For Orthogonal Model

Velocity of Tool relative to workpiece \( V \)

\( F_c \) Tangential ‘Cutting’ Force (67%)

\( F_r \) Radial Force (6%)

Longitudinal ‘Thrust’ Force (27%)

\( F_t \)

DIRECTION OF FEED

DIRECTION OF ROTATION

Note: For the 2D Orthogonal Mechanistic Model we will ignore the radial component
‘Facing’ Forces For Orthogonal Model

**Direction of Rotation**
- **$F_L$** Longitudinal Force
- **$F_C$** Tangential Force ‘Cutting’ Force

**Direction of Feed**
- **$F_R$** Radial Force ‘Thrust’ Force

**Note:** For the 2D Orthogonal Mechanistic Model we will ignore the Longitudinal component.
'Turning' Terminology

\[ N \text{ rpm} \]

\[ \phi D \]

\[ \text{Workpiece} \]

\[ d \text{ mm} \]

\[ \text{Tool} \]

\[ \text{feed (mm/rev)} \]

Standard Terms

- \( N \) is the speed in rpm
- \( D \) is the diameter of the workpiece
- \( f \) is the feed (linear distance/rev)
- \( d \) is the depth of cut
- \( V \) is the surface speed \( = \pi DN \)

Beware, for turning: In the generalized orthogonal model depth of cut (to) is \( f \) (the feed), and width of cut (w) is \( d \) (the depth of cut)
'Turning' Terminology

Standard Terms

N is the speed in rpm
D is the diameter of the workpiece
f is the feed (linear distance/rev)
d is the depth of cut
V is the surface speed
\[ V = \pi DN \]

Beware, for turning: In the generalized orthogonal model depth of cut (t) is f (the feed), and width of cut (w) is d (the depth of cut)
Orthogonal Cutting Model
(Simple 2D mechanistic model)

Mechanism: Chips produced by the shearing process along the shear plane
Cutting Ratio
(or chip thickness ratio)

As \( \sin \phi = \frac{t_0}{AB} \) and \( \cos(\phi - \alpha) = \frac{t_c}{AB} \),

Chip thickness ratio \( r = \frac{t_0}{t_c} = \frac{\sin \phi}{\cos(\phi - \alpha)} \).
Experimental Determination of Cutting Ratio

Shear angle $\phi$ may be obtained either from photo-micrographs or assume volume continuity (no chip density change):

Since $t_0w_0L_0 = t_cw_cL_c$ and $w_0=w_c$ (exp. evidence)

Cutting ratio $r = \frac{t_0}{t_c} = \frac{L_c}{L_0}$

i.e. Measure length of chips (easier than thickness)
Shear Plane Length
and Angle $\phi$

$\text{Shear plane length } AB = \frac{t_0}{\sin \phi}$

$\text{Shear plane angle } (\phi) = \tan^{-1} \left[ \frac{r \cos \alpha}{1 - r \sin \alpha} \right]$}

or make an assumption, such as $\phi$ adjusts to minimize cutting force: $\phi = 45^0 + \alpha/2 - \beta/2$ (Merchant)
Shear Velocity (Chip relative to workpiece)

\[ V_c = \text{Chip Velocity (Chip relative to tool)} \]

\[ V = \text{Cutting Velocity (Tool relative to workpiece)} \]

**From mass continuity:**

\[ Vt_0 = V_c t_c \]

\[ V_c = Vr \text{ and } V_c = V \frac{\sin \phi}{\cos(\phi - \alpha)} \]

**From the Velocity diagram:**

\[ V_s = V \frac{\cos \alpha}{\cos(\phi - \alpha)} \]
Cutting Forces

(2D Orthogonal Cutting)

Generally we know:
Tool geometry & type
Workpiece material

and we wish to know:
\( F \) = Cutting Force
\( F_c \) = Thrust Force
\( F_t \) = Friction Force
\( N \) = Normal Force
\( F_s \) = Shear Force
\( F_n \) = Force Normal
to Shear

Free Body Diagram
Force Circle Diagram

(Merchants Circle)
Results from Force Circle Diagram

(Merchant's Circle)

Friction Force \( F = F_c \sin \alpha + F_t \cos \alpha \)

Normal Force \( N = F_c \cos \alpha - F_t \sin \alpha \)

\( \mu = \frac{F}{N} \) and \( \mu = \tan \beta \) (typically 0.5 - 2.0)

Shear Force \( F_s = F_c \cos \phi - F_t \sin \phi \)

Force Normal to Shear plane \( F_n = F_c \sin \phi + F_t \cos \phi \)

\[ R = \sqrt{F_c^2 + F_t^2} = \sqrt{F_s^2 + F_n^2} = \sqrt{F^2 + N^2} \]
Forces on the Cutting Tool and the workpiece

- Importance: Stiffness of tool holder, stiffness of machine, and stiffness of workpiece must be sufficient to avoid significant deflections (dimensional accuracy and surface finish)
- Primary cause: Friction force of chip up rake face + Shearing force along shear plane
- Cutting speed does not effect tool forces much (friction forces decrease slightly as velocity increases; static friction is the greatest)
- The greater the depth of cut the greater the forces on the tool
- Using a coolant reduces the forces slightly but greatly increases tool life
Stresses

On the Shear plane:

Normal Stress = $\sigma_s = \text{Normal Force} / \text{Area} = \frac{F_n}{AB \cdot w} = \frac{F_n \sin \phi}{t_ow}$

Shear Stress = $\tau_s = \text{Shear Force} / \text{Area} = \frac{F_s}{AB \cdot w} = \frac{F_s \sin \phi}{t_ow}$

Note: $\tau_s = \tau_y = \text{yield strength of the material in shear}$

On the tool rake face:

$\sigma = \frac{N}{t_c \cdot w}$ (often assume $t_c = \text{contact length}$)

$\tau = \frac{F}{t_c \cdot w}$
Power

- Power (or energy consumed per unit time) is the product of force and velocity. Power at the cutting spindle:
  \[ \text{Cutting Power } P_c = F_c V \]
- Power is dissipated mainly in the shear zone and on the rake face:
  \[ \text{Power for Shearing } P_s = F_s V_s \]
  \[ \text{Friction Power } P_f = FV_c \]
- Actual Motor Power requirements will depend on machine efficiency \( E \) (%):
  \[ \text{Motor Power Required} = \frac{P_c}{E} \times 100 \]
Material Removal Rate (MRR)

\[
\text{Material Removal Rate (MRR)} = \frac{\text{Volume Removed}}{\text{Time}}
\]

Volume Removed = Lwt_o

Time to move a distance L = L/V

Therefore, \[
\text{MRR} = \frac{Lwt_o}{L/V} = Vwt_o
\]

MRR = Cutting velocity \times width of cut \times depth of cut
Specific Cutting Energy
(or Unit Power)

Energy required to remove a unit volume of material (often quoted as a function of workpiece material, tool and process):

\[ U_t = \frac{\text{Energy}}{\text{Volume Removed}} = \frac{\text{Energy per unit time}}{\text{Volume Removed per unit time}} \]

\[ U_t = \frac{\text{Cutting Power } (P_c)}{\text{Material Removal Rate (MRR)}} = \frac{F_c V}{V_{wt_o}} = \frac{F_c}{ wt_o} \]

Specific Energy for shearing \[ U_s = \frac{F_s V_s}{V_{wt_o}} \]

Specific Energy for friction \[ U_f = \frac{F V_c}{V_{wt_o}} = \frac{F_r}{ wt_o} \]
Specific Cutting Energy

Decomposition

1. Shear Energy/unit volume ($U_s$) (required for deformation in shear zone)
2. Friction Energy/unit volume ($U_f$) (expended as chip slides along rake face)
3. Chip curl energy/unit volume ($U_c$) (expended in curling the chip)
4. Kinetic Energy/unit volume ($U_m$) (required to accelerate chip)

$$U_t = U_s + U_f + U_c + U_m$$
Specific Cutting Energy
Relationship to Shear strength of Material

SHEAR ENERGY / UNIT VOLUME

Specific Energy for shearing \( U_s = \frac{F_s V_s}{V_{wt_o}} \)

\[ U_s = \frac{\tau_s \cos \alpha}{\sin \phi \cos (\phi - \alpha)} = \tau_s \gamma \]

FRICTION ENERGY / UNIT VOLUME

Specific Energy for friction \( U_f = \frac{F V_c}{V_{wt_o}} = \frac{F r}{w t_o} = \frac{F}{w t_c} = \tau \)

APPROXIMATE TOTAL SPECIFIC CUTTING ENERGY

\[ U_t = U_s + U_f = \tau_s \gamma + \tau = \tau y(1+\gamma) \]
Single point Cutting Tool

A *chip of material* is removed from the surface of the workpiece.

**Principal parameters:**
- the cutting speed, $v$
- the depth of cut, $w$ or $d$
- the feed, $f$.

*Time* requires to turn a cylindrical surface of length $L_w$.

$$ t = \frac{L_w}{fn_{nw}} $$

Where $n_{nw}$ is the number of revolutions of the workpiece per second.
The procedure to construct a merchants circle diagram

- Set up x-y axis labeled with forces, and the origin in the centre of the page. The cutting force (Fc) is drawn horizontally, and the tangential force (Ft) is drawn vertically. (Draw in the resultant (R) of Fc and Ft.
- Locate the centre of R, and draw a circle that encloses vector R. If done correctly, the heads and tails of all 3 vectors will lie on this circle.
- Draw in the cutting tool in the upper right hand quadrant, taking care to draw the correct rake angle (α) from the vertical axis.
- Extend the line that is the cutting face of the tool (at the same rake angle) through the circle. This now gives the friction vector (F).
- A line can now be drawn from the head of the friction vector, to the head of the resultant vector (R). This gives the normal vector (N). Also add a friction angle (β) between vectors R and N. Therefore, mathematically, R = Fc + Ft = F + N.
- Draw a feed thickness line parallel to the horizontal axis. Next draw a chip thickness line parallel to the tool cutting face.
- Draw a vector from the origin (tool point) towards the intersection of the two chip lines, stopping at the circle. The result will be a shear force vector (Fs). Also measure the shear force angle between Fs and Fc.
- Finally add the shear force normal (Fn) from the head of Fs to the head of R.
- Use a scale and protractor to measure off all distances (forces) and angles.
Relationship of various forces acting on the chip with the horizontal and vertical cutting force from Merchant circle diagram

Frictional Force System

\[ F = OA = CB = CG + GB = ED + GB \]

\[ \Rightarrow F = F_c \sin \alpha + F_t \cos \alpha \]

\[ N = AB = OD - CD = OD - GE \]

\[ \Rightarrow N = F_c \cos \alpha = F_t \sin \alpha \]

The coefficient of friction

\[ \mu = \tan \beta = \frac{F}{N} \]

Where \( \beta = \text{Friction angle} \)
Relationship of various forces acting on the chip with the horizontal and vertical cutting force from Merchant circle diagram

Shear Force System

\[ F_S = OA = OB - AB = OB - CD \]

\[ \Rightarrow F_S = F_C \cos \phi - F_t \sin \phi \]

\[ F_N = AE = AD + DE = BC + DE \]

\[ \Rightarrow F_N = F_C \sin \phi + F_t \cos \phi \]

Also:

\[ F_N = F_S \tan(\phi + \beta - \alpha) \]
Power required in Metal cutting

The Power consumed/ work done per sec in cutting: 

\[ P_C = F_C \times v_C \]

The Power consumed/ work done per sec in shear: 

\[ P_s = F_s \times v_s \]

The Power consumed/ work done per sec in friction: 

\[ P_f = F \times v_f \]

The total Power required:

\( P = \) Power supplied by the motor

\( \Rightarrow P = \) Work consumed in cutting per sec + work spent in feeding per sec

\( \Rightarrow P = F_c \times v_c + F_f \times feed\ velocity \)

In comparison to the cutting velocity the feed velocity is very nominal. Similarly \( F_c \) is very small compared to \( F_c \). So the work spent in feeding can be considered negligible.

Therefore, total power required in cutting 

\[ P = P_c = P_s + P_f \]
Specific Energy

Specific Energy, $u_t$, is defined as the total energy per unit volume of material removed.

$$u_t = \frac{F_C v_c}{w t_0 v_c} = \frac{F_C}{w t_0}$$

Therefore is simply the cutting force to the projected area of cut.

If $u_t$ and $u_s$ be specific energy for friction and specific energy for shearing, then

$$u_t = u_f + u_s = \frac{F v_f}{w t_0 v_c} + \frac{F_s v_s}{w t_0 v_c} = \frac{F_r}{w t_0} + \frac{F_s v_s}{w t_0 v_c}$$

As the rake angle increases, the frictional specific energy remains more or less constant, whereas the shear specific energy rapidly reduced.