First Semester B.E. Degree Examination, May/June 2010

Engineering Mathematics - I

Time: 3 hrs.

Note: 1. Answer any FIVE full questions, choosing at least two from each part.
2. Answer all objective type questions only on OMR sheet page 5 of the Answer Booklet.
3. Answer to objective type questions on sheets other than OMR will not be valued.

PART – A

1. a. i) The \( n \)th derivative of \( \frac{1}{x^p} \) is

A) \( \frac{(-1)^p (p+n)!}{(p-1)! x^{p+n}} \) 
B) \( \frac{(-1)^p (p+n-1)!}{(p-1)! x^{p+n}} \)
C) \( \frac{(-1)^p (p+n-1)!}{(p-1)! x^{p+n}} \) 
D) \( \frac{(-1)^p (p+n-1)!}{p! x^p} \)

ii) The \( n \)th derivative of \( e^x \) is

A) \( a^n e^{ax} \) 
B) \( ae^x \) 
C) \( a^2 e^x \) 
D) \( e^x \)

iii) The angle between radius vector and tangent is

A) \( \tan \phi = \frac{d\theta}{dr} \) 
B) \( \tan \phi = r^2 \frac{d\theta}{dr} \) 
C) \( \tan \phi = \frac{1}{r} \frac{d\theta}{dr} \) 
D) \( \tan \phi = \frac{dr}{d\theta} \)

iv) The curve \( r = \frac{a}{1 + \cos \theta} \) intersect orthogonally with the following curve:

A) \( r = \frac{b}{1 - \cos \theta} \) 
B) \( r = \frac{b}{1 + \sin \theta} \) 
C) \( r = \frac{b}{1 + \sin^2 \theta} \) 
D) \( r = \frac{b}{1 + \cos^2 \theta} \)

b. Find the \( n \)th derivation of \( y = \sin h 2x \sin 4x \).

c. If \( y = \sin h (m \log(x + \sqrt{x^2 + 1})) \), prove that \( (x^2 + 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0 \).

d. Find the pedal equation of the curve \( r = a \cos (m \theta) + a \sin (m \theta) \).

2. a. i) If \( u = \log \left( \frac{x^2}{y} \right) \), then \( x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \) is equal to

A) 2u 
B) 3u 
C) u 
D) 1

ii) If \( u = x^3 + y^3 \), then \( \frac{\partial^3 u}{\partial x^3 \partial y} \) is equal to

A) -3 
B) 3 
C) 0 
D) 3x + 3y

iii) If \( x = r \cos \theta \), \( y = r \sin \theta \), then \( J\frac{\partial (x,y)}{\partial (r,\theta)} \) is equal to

A) 1 
B) r 
C) \( \frac{1}{r} \) 
D) 0

iv) If an error of 1% is made in measuring its length and breadth, the percentage error in the area of a rectangle is

A) 0.2% 
B) 0.02% 
C) 2% 
D) 1% (04 Marks)

b. If \( z = e^{ax+by} + (ax - by) \), prove that \( b \frac{\partial z}{\partial x} + a \frac{\partial z}{\partial y} = 2abz \).

(04 Marks)
4. Find the surface area of the solid got by revolving the arch of the cycloid \( x = a(t + \sin t), \)
\( y = a(t + \cos t) \) about the base.  
(06 Marks)

d. Evaluate \( \int_{\alpha}^{\pi} \arctan \left( \frac{\tan^{-1} \alpha}{x(1 + x^2)} \right) \, dx \) where \( \alpha \geq 0 \) using the rule of differentiation under the integral sign.  
(06 Marks)

PART – B

5. a. i) The order of the differential equation \( \left( \frac{dy}{dx} \right)^2 - 5 \frac{dy}{dx} + 4y = 0 \) is
   A) 2 \hspace{1cm} B) 0 \hspace{1cm} C) 3 \hspace{1cm} D) 1

   ii) The integrating factor of the differential equation \( \frac{dy}{dx} + y \cos x = \frac{\sin 2x}{2} \) is
   A) \( e^{\sin^2 x} \) \hspace{1cm} B) \( e^{\sin x} \) \hspace{1cm} C) \( e^{\sin x} \) \hspace{1cm} D) \( \sin x \)

   iii) The solution of the differential equation \( \frac{dy}{dx} = \frac{y}{x} - \csc x \) is
   A) \( \cos \left( \frac{y}{x} \right) - \log x = c \) \hspace{1cm} B) \( \cos \left( \frac{y}{x} \right) + \log x = c \)
   C) \( \cos^2 \left( \frac{y}{x} \right) + \log x = c \) \hspace{1cm} D) \( \cos^2 \left( \frac{y}{x} \right) - \log x = c \)

   iv) By replacing \( \frac{dr}{d\theta} \) by \( -r^2 \frac{dr}{d\theta} \) in the differential equation \( f(r, \theta, -r^2 \frac{dr}{d\theta}) = 0 \), we get the differential equation of __________.  
   A) Orthogonal trajectory \hspace{1cm} B) Polar trajectory \hspace{1cm} C) Parametric trajectory \hspace{1cm} D) None of these.  
   (04 Marks)

b. Solve: \( (1-x^2) \frac{dy}{dx} - xy = 1 \).  
   (04 Marks)

c. Solve: \( xdx + ydy + \frac{x dy - y dx}{x^2 + y^2} = 0 \).  
   (06 Marks)

d. Find the orthogonal trajectories of the family of curves \( r = 2a(\cos \theta + \sin \theta) \) where \( a \) is a parameter.  
   (06 Marks)

6. a. i) The series \( \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \ldots \ldots \) converges if
   A) \( p > 0 \) \hspace{1cm} B) \( p < 1 \) \hspace{1cm} C) \( p > 1 \) \hspace{1cm} D) \( p \leq 1 \)

   ii) \( \sum \sin \left( \frac{1}{n} \right) \) is
   A) convergent \hspace{1cm} B) divergent \hspace{1cm} C) oscillatory \hspace{1cm} D) none of these

   iii) The convergence of the series \( \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{5}} + \ldots \ldots \)
   A) Leibnitz test \hspace{1cm} B) Raabe's test \hspace{1cm} C) Ratio test \hspace{1cm} D) Cauchy's root test

   iv) If a series \( \sum y_n \) is such that \( S_n \) does not tend to unique limit as \( n \to \infty \), we say that the series \( \sum y_n \) is
   A) convergent \hspace{1cm} B) divergent \hspace{1cm} C) oscillatory \hspace{1cm} D) none of these  
   (04 Marks)

b. Determine the nature of the series \( \sum \frac{1}{\sqrt{n^2 + 1 - n}} \).  
   (04 Marks)

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