First Semester B.E. Degree Examination, June/July 2015
Engineering Mathematics – I

Time: 3 hrs.
Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing at least two from each part.
2. Answer all objective type questions only on OMR sheet page 5 of the answer booklet.
3. Answer to objective type questions on sheets other than OMR will not be valued.

PART – A

1 a. Choose the correct answers for the following: (04 Marks)

i) If \( y = x^{2n} \), \( y_{n+1} \) is equal to __________
   A) \( 0 \)  B) \( \frac{n!}{(2n)!} x^n \)  C) \( \frac{2n!}{(n-1)!} x^{n-1} \)  D) \( \frac{2n!}{(n-1)!} x^{n-1} \)

ii) If \( y = x^n \log x \) then by Leibnitz theorem \( xy_{n+1} = ________ \)
    A) \( (n-1)! \)  B) \( (n+1)! \)  C) \( n! \)  D) \( 0 \)

iii) If \( f(x) = \sqrt{x} \), \( g(x) = \frac{1}{\sqrt{x}} \) then by Cauchy’s mean value theorem \( C = ________ \)
    A) \( \sqrt{a-b} \)  B) \( \sqrt{a+b} \)  C) \( \sqrt{ab} \)  D) \( \frac{a}{b} \)

iv) By Maclaurin’s series \( 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots \) is equal to,
   A) \( e^x \)  B) \( \sin x \)  C) \( \cos x \)  D) \( \log(1+x) \)

b. If \( y = \log(x + \sqrt{1 + x^2}) \) Prove that \( (1 + x^2)y_{n+2} + 2(n + 1)xy_{n+1} + n^2 y_n = 0 \) (04 Marks)

c. State Lagrange’s mean value theorem, and find the number ‘C’ in [0, 4] when \( f(x) = (x-1)(x-2)(x-3). \) (06 Marks)

d. Expand \( \log_e x \) in the powers of \( x-1 \) and hence evaluate \( \log_e(1.1) \) by taking up to 4th degree terms. (06 Marks)

2 a. Choose the correct answers for the following: (04 Marks)

i) \( \lim_{x \to \frac{\pi}{2}} \frac{\sec^2 x - \tan x}{\sin x - \cos x} = \) __________
   A) \( 0 \)  B) \( 1 \)  C) \( \pi/2 \)  D) \( \pi \)

ii) The angle between radius vector and the tangent to the curve \( r = \sin \theta + \cos \theta \) is __________
    A) \( \frac{\pi}{4} - \theta \)  B) \( \frac{\pi}{4} + \theta \)  C) \( \frac{\pi}{2} + \theta \)  D) \( \frac{\pi}{2} - \theta \)

iii) The derivative of arc length for the curve \( x = f(y) \) is __________
     A) \( \sqrt{1 + y^2_1} \)  B) \( \sqrt{x^1_1 + y^2_1} \)  C) \( \sqrt{1 + x^2} \)  D) \( \sqrt{1 - y^2_1} \)

iv) The radius of curvature of the curve \( 2ap^2 = r^3 \) is __________
   A) \( \frac{3}{2} \sqrt{2ar} \)  B) \( \frac{3}{2} \sqrt{ar} \)  C) \( \frac{2}{3} \sqrt{ar} \)  D) \( \frac{4ap}{3r} \)

b. Evaluate \( \lim_{x \to 0} \left( \frac{1}{x^2} - \frac{1}{\sin^2 x} \right) \). (04 Marks)

c. Find the angle of intersection between the curves \( r^3 \sin 2\theta = 4 \) and \( r^2 \sec 2\theta = 16 \). (06 Marks)

d. Find the radius of curvature at any point t on the curve \( x = a \left( \cos t + \log \tan \frac{t}{2} \right), y = a \sin t \). (06 Marks)
3 a. Choose the correct answers for the following:  
   (04 Marks)
   i) If \( u = x^2 + y^2 + z^2 \) then \( \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^3 u}{\partial z^3} = \) 
      A) \( 2(x + y) \)  
      B) \( 2(x + 1) \)  
      C) \( 2(x + z) \)  
      D) \( 2(y + z) \)
   ii) For \( u = x(1 - y), V = xy \) the value of Jacobian is, 
      A) \( x \)  
      B) \( x^2 \)  
      C) \( xy \)  
      D) \( \frac{x}{y} \)
   iii) In the Taylor’s expansion of \( f(x, y) = xy^2 + \cos(xy) \) about \( \left( 1, \frac{\pi}{2} \right) \) the value of the derivative \( \frac{\partial^2 f}{\partial x \partial y} \) at the given point is 
      A) \( \pi + 1 \)  
      B) \( \pi + 2 \)  
      C) \( \pi - 1 \)  
      D) \( \pi - 2 \)
   iv) For \( f(x, y) = x^3y^2(1 - x - y) \), one set of stationary values are, 
      A) \( \left( \frac{1}{2}, \frac{1}{2} \right) \)  
      B) \( \left( \frac{1}{3}, \frac{1}{3} \right) \)  
      C) \( \left( \frac{1}{3}, \frac{1}{4} \right) \)  
      D) \( \left( \frac{1}{2}, \frac{1}{3} \right) \)

b. If \( u = f(y - z, z - x, x - y) \), find the value of \( \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} \)  
   (04 Marks)

c. If \( u = x + y + z, \ u v = y + z, \ u v w = z \) then find the value of \( \frac{\partial (x, y, z)}{\partial (u, v, w)} \).  
   (06 Marks)

d. A rectangular box open at the top is to have a volume of 32 cubic units, find the dimensions of the box requiring least material for its construction.  
   (06 Marks)

4 a. Choose the correct answers for the following:  
   (04 Marks)
   i) The representation \( i \frac{\partial f}{\partial x} + j \frac{\partial f}{\partial y} + k \frac{\partial f}{\partial z} \) is for 
      A) \( \nabla \cdot f \)  
      B) \( \nabla \times f \)  
      C) \( \nabla^2 f \)  
      D) \( \nabla f \)
   ii) If \( \text{div} \ V = 0 \) when \( V \) is the volume then such a point of function is called 
      A) Rotational  
      B) Irrotational  
      C) Solenoidal  
      D) Orthogonal
   iii) \( \text{curl (grade)} \) is denoted by, 
      A) \( \nabla \cdot (\nabla \phi) \)  
      B) \( \nabla \times (\nabla \cdot \phi) \)  
      C) \( \nabla \times (\nabla \phi) \)  
      D) \( \nabla \cdot (\nabla \phi) \)
   iv) If \( \hat{e}_1, \hat{e}_2, \hat{e}_3 \) are the base vectors then the positive numbers \( h_1, h_2, h_3 \) are called the, 
      A) volume factors  
      B) scale factors  
      C) area factors  
      D) acceleration factors

b. A vector field is given by, \( \vec{F} = (x^2 - y^2 + x)i - (2x + y)j \), show that the field is irrotational.  
   (04 Marks)

c. Prove that \( \text{div} (\text{curl} \ A) = 0 \).  
   (06 Marks)

d. Prove that cylindrical coordinate system is orthogonal.  
   (06 Marks)

5 a. Choose the correct answers for the following:  
   (04 Marks)
   i) If \( I(\alpha) = \int_0^1 x^\alpha \frac{dx}{\log x} \) then \( \frac{dI(\alpha)}{d\alpha} = \) 
      A) \( \frac{1}{1 - \alpha} \)  
      B) \( \frac{1}{1 + \alpha} \)  
      C) \( \frac{1}{1 + \alpha^2} \)  
      D) \( \frac{1}{1 - \alpha^2} \)

PART - B
ii) The value of \( \int_0^\frac{\pi}{2} \sin^4 x \, dx \) is ____________

A) \( \frac{3\pi}{8} \)  
B) \( \frac{4\pi}{5} \)  
C) \( \frac{5\pi}{8} \)  
D) \( \frac{6\pi}{7} \)

iii) The volume generated by revolving \( y = f(x) \) between \( x = a \) and \( x = b \) is \( V = \) ____________

A) \( \int_a^b y^2 \, dx \)  
B) \( \int_a^b \pi y^2 \, dx \)  
C) \( \int_a^b \pi y \, dx \)  
D) \( \int_a^b \pi x^2 \, dx \)

d. Find the entire length of the curve \( x^{\frac{3}{5}} + y^{\frac{3}{5}} = a^{\frac{3}{5}} \).

iv) Special points on \( x \) and \( y \) axes are ____________ for the curve \( x^{\frac{3}{5}} + y^{\frac{3}{5}} = a^{\frac{3}{5}} \).

A) \( \pi a^2 b \)  
B) \( \pi ab^2 \)  
C) \( \pi a^2 b^2 \)  
D) \( \pi ab \)

b. Evaluate \( \int_0^\infty \frac{\tan^{-1} ax}{x(1 + x^2)} \, dx \) using differentiation under integral sign. (04 Marks)

c. Evaluate \( \int_0^{\frac{2a}{2}} x^2 \sqrt{2ax - x^2} \, dx \). (06 Marks)

d. Find the entire length of the curve \( x^{\frac{3}{5}} + y^{\frac{3}{5}} = a^{\frac{3}{5}} \). (06 Marks)

6 a. Choose the correct answers for the following:

i) Solution of \( (2x + 1) + (2y + 1) \frac{dy}{dx} = 0 \) is ____________

A) \( x^2 + y^2 + x + y = C \)  
B) \( 2x^2 + 2y^2 + x + y = C \)  
C) \( \frac{x^2}{2} + \frac{y^2}{2} + x + y = C \)  
D) \( x^2 + y^2 + 2x + 2y = C \)

ii) For the linear differential equation \( \frac{dx}{dy} + P(y)x = Q(y) \) the integrating factor is ____________

A) \( e^{\int P(x) \, dx} \)  
B) \( e^{\int P(y) \, dy} \)  
C) \( e^{\int P(y) \, dy} \)  
D) \( e^{\int Q(y) \, dy} \)

iii) In the exact differential equation, choosing \( \frac{1}{M} \left( \frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} \right) \) denotes,

A) function of \( x \) alone  
B) function of \( y \) alone  
C) function of \( x \) and \( y \)  
D) function of \( \frac{x}{y} \)

iv) In the orthogonal trajectory of \( r = f(\theta) \) we replace \( \frac{dr}{d\theta} \) by ____________

A) \( -r \frac{dr}{d\theta} \)  
B) \( -r^2 \frac{d\theta}{dr} \)  
C) \( -r \frac{d\theta}{dr} \)  
D) \( -r^2 \frac{d\theta}{dr} \)

b. Solve \( (x + 2y)(dx - dy) = dx + dy \). (04 Marks)

c. Solve \( (x^2 + y^3 + 6x)dx + y^2 xdy = 0 \). (06 Marks)

d. Find the orthogonal trajectories of the curve \( r = 4a \sec \theta \tan \theta \) with \( a \) as the parameter. (06 Marks)
7. a. Choose the correct answers for the following: (04 Marks)
   i) If the elements in a square matrix below the main diagonal are zero then it is called ________.
      A) Orthogonal matrix  B) Singular matrix
      C) Lower triangular matrix  D) Upper triangular matrix
   ii) The rank of the matrix \( A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \) is ________,
      A) 0  B) 1  C) 2  D) 3
   iii) The system of equations are said to be consistent when,
      A) \( R(A) = R(A : B) \)  B) \( R(A) = R(A : B) \)  C) \( R(A) < R(A : B) \)  D) \( R(A) > R(A : B) \)
   iv) In Gauss Jordan method the coefficient matrix is reduced to ________,
      A) diagonal matrix  B) Upper triangular matrix
      C) null matrix  D) non-diagonal matrix
b. Test for consistency and solve the system of equations \( x + 2y + 2z = 5 \), \( 2x + y + 3z = 6 \), \( 3x - y + 2z = 4 \) and \( x + y + z = -1 \). (04 Marks)
c. Find the rank of the matrix \( A = \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix} \). (06 Marks)
d. Solve the system of equations by Gauss-Jordan method. \( x + y + z = 9 \), \( x - 2y + 3z = 8 \) and \( 2x + y - z = 3 \). (06 Marks)

8. a. Choose the correct answers for the following: (04 Marks)
   i) The Eigen values of the matrix \( A = \begin{bmatrix} -3 & 8 \\ -2 & 7 \end{bmatrix} \) are ________.
      A) 5, -1  B) 5, 1  C) 5, 2  D) 5, -2
   ii) A square matrix \( A \) of order \( n \) is called similar to a square matrix \( A \) of order \( n \) if \( A = PAP^{-1} \).
      A) \( PA^{-1}P \)  B) \( P^{1}AP \)  C) \( P^{-1}A^{1}P \)  D) \( PA^{1}P^{1} \)
   iii) A homogeneous expression of second degree in any number of variables is called ________.
      A) Orthogonal form  B) diagonal form  C) triangular form  D) quadratic form
   iv) If the eigen vector is \((1, 1, 1)\) then its normalized form is ________.
      A) \( \left( \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right) \)  B) \( \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) \)
      C) \( (1,1,1) \)  D) \( (\sqrt{3}, \sqrt{3}, \sqrt{3}) \)
   b. Find the eigen values and the eigen vectors of the matrix \( \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \). (04 Marks)
c. Reduce the quadratic form, \( 3x^2 + 5y^2 + 3z^2 - 2xy + 2zx - 2xy \) to the canonical form, specify the matrix of transformation. (06 Marks)
d. Find the nature of the following quadratic form, \( x^2 + 5y^2 + z^2 + 2xy + 2yz + 6zx \) (06 Marks)

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