Second Semester B.E. Degree Examination, June/July 2016
Engineering Mathematics – II

Time: 3 hrs.
Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing at least two from each part.
   2. Answer all objective type questions only on OMR sheet page 5 of the answer booklet.
   3. Answer to objective type questions on sheets other than OMR will not be valued.

PART – A

1. a. Choose correct answers for the following:
   (04 Marks)
   
   i) If $P = \frac{dy}{dx}$, then the differential equation of the first order and higher degree is of the form,
   A) $f(y, P) = 0$  B) $f(x, P) = 0$  C) $f(x, y) = 0$  D) $f(x, y, P) = 0$
   
   ii) A differential equation of the form, $P^2 + P - 6 = 0$ has the general solution,
   A) $(y + 3x)(y - 2x) = 0$  B) $(y + 3x - c)(y - x - c) = 0$
   C) $(y + 3x - c)(y - 2x - c) = 0$  D) $(y + x - c)(y - 2x - c) = 0$
   
   iii) If the given differential equation is solvable for $y$, takes the form,
   A) $y = f\left(\sqrt{y} \right)$  B) $y = f(x, P)$  C) $x = f(y, P)$  D) $y = f\left(\frac{P}{x} \right)$
   
   iv) The general solution of the equation, $P = \log_e(Px - y)$ is,
   A) $C = \log_e(Cx - y)$  B) $y = \log_e(Cx - y)$
   C) $x = \log_e(Cx - y)$  D) None of these

b. Solve $y\left(\frac{dy}{dx}\right)^2 + (x - y)\frac{dy}{dx} - x = 0$ by solving for $P$.
   (04 Marks)

c. Solve $y - 2px = \tan^{-1}(xp^2)$ by solving for $y$.
   (06 Marks)
d. Solve $x^2(y' \ p(x) = yp^2)$, using the substitution $X = x^2$, $Y = y'$.
   (06 Marks)

2. a. Choose correct answers for the following:
   (04 Marks)
   
   i) A solution of the following differential equation, $(D^2 - 5D + 6)y = 0$ is,
   A) $y = C_1e^{2x} + C_2e^{3x}$  B) $y = C_1e^{2x} + C_2e^{-3x}$
   C) $y = C_1e^{-2x} + C_2e^{3x}$  D) $y = C_1e^{2x} + C_2e^{4x}$
   
   ii) Which one of the following solution does not satisfy the differential equation, $\frac{d^2y}{dx^2} - y = 0$?
   A) $e^{-x}$  B) $e^x$  C) $e^x \sin \frac{\sqrt{3}}{2} x$  D) $e^x \cos \frac{\sqrt{3}}{2} x$
   
   iii) If $m = 2$ is a repeated root and $m = -1$ is another root of the auxiliary equation of a homogeneous differential equation with constant coefficients. The differential equation is,
   A) $(D^2 + 3D^2 + 4)y = 0$  B) $(D^2 + 3D^2 - 4)y = 0$
   C) $(D^2 - 3D^2 + 4)y = 0$  D) $(D^2 - 3D^2 - 4)y = 0$
   
   iv) The particular integral for the differential equation, $(D^2 + 4D + 3)y = 3e^{2x}$ is,
   A) $\frac{1}{15}e^{2x}$  B) $\frac{1}{5}e^{2x}$  C) $3e^{2x}$  D) $5e^{2x}$

b. Solve $(D - 2)^2y = 8(e^{2x} + \sin 2x)$.
   (04 Marks)

c. Solve $(D^2 - 1)y = x \sin x + (1 + x^2)e^x$.
   (06 Marks)

d. Solve the simultaneous differential equations, $\frac{dx}{dt} = 5x + y$, $\frac{dy}{dt} = y - 4x$.
   (06 Marks)

VTU1-4/6/2016
3  a. Choose correct answers for the following:
   i) The Wronskian of $e^{-x}$ and $xe^{-x}$ is,
      A) $e^{-2x}$                           B) $(1-x)e^{-2x}$
      C) $-e^{-2x}$                          D) $(x-1)e^{-2x}$
   ii) The complementary function of the differential equation, $x^2y'' - xy' + y = \log_e x$ is,
       A) $\log_e x + 2$                       B) $\log x$
       C) $x \log_e x + 2$                D) $\frac{x}{2}(\log_e x)^2$
   iii) If $D = \frac{d}{dx}$ and $t = \log_e (x+1)$, then the differential equation,
       $(1+x)^2 y'' + (1+x)y' + y = 2\sin \log(1+x)$ becomes,
       A) $(D^2 + 1)y = 2\sin t$           B) $(D^2 + 1)y = 2\cos t$
       C) $(D^2 - 1)y = 2\sin t$           D) $(D^2 - 1)y = 2\cos t$
   iv) Series solution is a regular singularity of the equation $P_0y'' + P_1y' + P_2y = 0$ when
       A) $x > 0$                           B) $x < 0$
       C) $x = 0$                           D) $x \neq 0$

b. Solve $(D^2 + 1)y = \csc x$ by the method of variation of parameters.
   c. Solve $(1 + x^2)\frac{d^2y}{dx^2} + (1 + x)\frac{dy}{dx} + y = 4 \cos \log_e (1 + x)$.
   d. Find the series solution of the differential equation, $\frac{d^2y}{dx^2} + xy = 0$.

4  a. Choose correct answers for the following:
   i) The order of the partial differential equation by eliminating $f$ from $z = f(x^2 + y^2)$ is,
      A) Second                           B) Third
      C) Zero                             D) First
   ii) The solution of the partial differential equation $x \frac{\partial z}{\partial x} = 2x + y$ is,
       A) $z = x^2 + xy + f(y)$            B) $z = 2x + y \log_e x + f(x)$
       C) $z = 2x + xy + f(y)$            D) $z = 2x + y \log_e x + f(y)$
   iii) The equation of the form $Pp + Qq = R$ is called,
       A) Legendre’s equation               B) Cauchy’s equation
       C) Euler’s equation                   D) Lagrange’s linear equation.
   iv) By the method of separation of variables the trial solution of $\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$ is,
       A) $u = X(x)Y(y)$                   B) $u = \frac{X(x)}{Y(y)}$
       C) $u = \frac{X(x)}{T(t)}$          D) $u = X(x)T(t)$

b. From the partial differential equation by eliminating the arbitrary function from $z = y^2 + 2f\left(\frac{1}{x} + \log_e y\right)$.

   c. Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$.
   d. Using the method of separation of variables, solve $3 \frac{\partial u}{\partial x} + z \frac{\partial u}{\partial y} = 0$, $u(x,0) = 4e^{-x}$

5  a. Choose correct answers for the following:
   i) The value of $\int_0^\pi \int_0^\pi \sin(x + y)dx dy$ is,
      A) 0                           B) 2
      C) $\pi$                       D) $\frac{\pi}{2}$
ii) The double integral \( \int_R r \, dr \, d\theta \) gives,

A) Volume of region \( R \)  
B) Area of region \( R \)
C) Surface area of region \( R \)  
D) Density of region \( R \).

iii) The value of \( \int_0^\infty e^{-x^2} \, dx \) is,

A) \( \sqrt{\pi} \)  
B) \( \frac{\sqrt{\pi}}{2} \)  
C) \( \pi \)  
D) \( \frac{\pi}{2} \)

iv) The value of \( \int_0^\frac{\pi}{2} \sqrt{\cot \theta} \, d\theta \) is,

A) \( \frac{\pi}{4} \)  
B) \( \frac{\pi \sqrt{2}}{2} \)  
C) \( \frac{\pi}{2} \)  
D) \( \frac{\pi \sqrt{2}}{4} \)

b. Evaluate \( \int_0^1 \int_0^1 xe^{-x/y} \, dy \, dx \) by changing the order of integration. (04 Marks)

c. Evaluate \( \int_0^1 \int_0^1 \int_0^1 \frac{1}{(x+y+z+1)} \, dz \, dy \, dx \). (06 Marks)

d. Show that \( \int_0^1 \int_0^1 \int_0^1 \frac{1}{\sqrt{1-x^4}} \, dx \, dz \, dy = \frac{\pi}{4} \). (06 Marks)

6 a. Choose correct answers for the following:

i) \( \oint_C \vec{F} \, d\vec{r} \) is the independent of the path joining any two points if and only if it is,

A) Irrotational field  
B) Rotational field  
C) Solenoidal field  
D) None of these

ii) If \( S \) is a closed surface enclosing a volume \( V \) and if \( \vec{F} = x \, \hat{i} + y \, \hat{j} + 2 \, \hat{k} \) then \( \iint_S \vec{F} \cdot d\vec{S} \) is,

A) 3V  
B) 2V  
C) V  
D) 4V

iii) The total work done in moving a particle in a force field \( \vec{F} = 3xy \, \hat{i} - 5yz \, \hat{j} + 10xz \, \hat{k} \) along the curve \( x = t^2 + 1, y = 2t, z = t^3 \) from \( t = 1 \) to \( t = 2 \) is,

A) 18  
B) 120  
C) 360  
D) 303

iv) Stokes theorem connects,

A) a line integral and a surface integral  
B) a surface integral and a volume integral  
C) a line integral and a volume integral  
D) None of these

b. Apply Green's theorem to evaluate \( \int_C (xy + y^2) \, dx + x^2 \, dy \), where \( C \) is bounded by \( y = x \) and \( y = x^2 \). (04 Marks)

c. Verify Stoke's theorem for \( \vec{F} = (x^2 + y^2) \, \hat{i} - 2xy \, \hat{j} \) taken around the rectangle bounded by the lines, \( x = \pm a, y = 0, y = b \). (06 Marks)

d. Using divergence theorem evaluate, \( \iint_S \vec{F} \cdot d\vec{S} \), where \( \vec{F} = yz \, \hat{i} + zx \, \hat{j} + xy \, \hat{k} \) and \( S \) is the surface of the sphere \( x^2 + y^2 + z^2 = a^2 \) in the first octant. (06 Marks)

7 a. Choose correct answers for the following:

i) If \( L[f(t)] = F(s) \), then \( L[e^{-at} f(t)] \) is,

A) \( -aF(s) \)  
B) \( F(s-a) \)  
C) \( e^{-as} F(s) \)  
D) \( F(s+a) \)

(04 Marks)
ii) The Laplace transform of $\sqrt{t}$ is, 
A) $\sqrt{\frac{\pi}{S}}$  
B) $\frac{1}{2} \sqrt{\frac{\pi}{S}}$  
C) $\sqrt{\frac{\pi}{S^2}}$  
D) does not exist

iii) $L\left(\int_0^t \cos t \, dt\right) =$ 
A) $\frac{s}{s^2 + 1}$  
B) $\frac{1}{s^2 + 1}$  
C) $\frac{1}{s^2 - 1}$  
D) $\frac{s}{s^2 - 1}$

iv) The Laplace transform of $\sin 2t \left(t - \frac{\pi}{4}\right)$ is, 
A) $e^{-\frac{s\pi}{4}}$  
B) $e^{-\frac{s\pi}{2}}$  
C) $e^{-\frac{s\pi}{2}}$  
D) $e^{\frac{s\pi}{2}}$

b. Find the value of $\int_0^\infty t e^{-2t} \cos t \, dt$. (04 Marks)

c. Find the Laplace transform of $f(t) = \begin{cases} E, & 0 < t < \frac{a}{2} \\ -E, & \frac{a}{2} < t < a \end{cases}$, where $f(t) = f(t + a)$. (06 Marks)

d. Express $f(t)$ in terms of unit step function and hence find the Laplace transform given that, 
\[
f(t) = \begin{cases} 1, & 0 < t \leq 1 \\ t, & 1 < t < 2 \\ t^2, & t > 2 \end{cases}
\] (06 Marks)

8 a. Choose correct answers for the following : (04 Marks)

i) $L^{-1}\left\{\frac{1}{s^n}\right\}$ is possible only when $u$ is, 
A) Negative integer  
B) Positive integer  
C) Negative rational  
D) None of these

ii) $L^{-1}\left\{\frac{1}{(s+3)^2}\right\} =$ 
A) $e^{-3t} t^4$  
B) $e^{-3t} t^3$  
C) $e^{-3t} t^2$  
D) $e^{-3t} t^4$  

iii) $L^{-1}\left\{\frac{1}{s}\right\} =$ 
A) $t$  
B) $\frac{1}{t}$  
C) $1$  
D) $u(t)$

iv) $L^{-1}\left\{\frac{se^{-\pi s}}{s^2 + a^2}\right\} =$ 
A) $\cos 3tu(t - \pi)$  
B) $-\cos 3tu(t - \pi)$  
C) $\frac{1}{3}\cos 3tu(t - \pi)$  
D) None of these

b. Find $L^{-1}\left\{\log\frac{s^2 + b^2}{s^2 + a^2}\right\}$. (04 Marks)

c. Find $L^{-1}\left\{\frac{1}{(s^2 + a^2)^2}\right\}$, by using convolution theorem. (06 Marks)

d. Solve by Laplace transform method, given $y'' - 6y' + 9y = t^2 e^{3t}$ and $y(0) = 2$, $y'(0) = 6$ (06 Marks)